

One center, one axis: an exact alignment test for off-center observers

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Consider a complete physical configuration that is spherically symmetric about a single center O , together with an observer at $M \neq O$ whose velocity has no component transverse to OM . Rotation covariance then implies an exact statement: every scalar sky map is axisymmetric about OM , so $a_{\ell m} = 0$ for $m \neq 0$ in the aligned frame, and every covariantly extracted nonzero dipole is parallel or antiparallel to the same axis. No field equation, flat-space approximation, or perturbative expansion is required. We complement this symmetry theorem with three local depth laws on a spatial slice. A line-of-sight integral of a smooth scalar has antipodal odd part $\frac{1}{2}s^2 \nabla F(M) \cdot \hat{n} + O(s^4)$; flux-limited counts have leading odd part $\frac{1}{4}s^4 \nabla \nu(M) \cdot \hat{n} + O(s^6)$; and an endpoint-potential redshift has leading odd part $-s \nabla \ln N(M) \cdot \hat{n} + O(s^3)$. These scalings distinguish geometric channels from a depth-independent kinematic dipole. We also give an exact negative result: a centered static isothermal emitting surface produces no temperature dipole for an off-center observer in a radial static potential. The resulting falsification test is simple: independently established non-kinematic residual dipoles attributed to one spherical structure must be collinear, while their depth dependence constrains the physical channel.

I. QUESTION AND EXACT SYMMETRY RESULT

The Copernican principle disfavors a privileged cosmic position, while observational tests ask whether residual sky statistics remain isotropic after kinematic, selection, and local-structure effects are treated. Off-center observers in spherically symmetric cosmologies have therefore been studied both as concrete models and as tests of the Copernican assumption [1–3]. The purpose here is narrower: before choosing any dynamics, what must be true if several residual observables are attributed to one spherically symmetric cosmic structure?

Let (Σ, g) be a smooth three-dimensional Riemannian spatial slice, $O \in \Sigma$ a proposed center, and $M \in \Sigma$ the observer at $r_o = d_g(O, M) > 0$. Let $\hat{e}_r$ be the outward radial unit vector at M , so $-\hat{e}_r$ points toward O . By the *complete configuration* $\mathcal{C}$ we mean everything entering the construction of the observable: spatial geometry, matter and radiation fields, physical boundaries, propagation law, and, for a deep data set, the relevant radial history.

Theorem (single-center alignment). Suppose $\mathcal{C}$ is invariant under $SO(3)$ rotations about O , and the observer at M is at rest or moves parallel to OM . Let $\mathcal{A}_{\mathcal{C},M}(\hat{n})$ be any sky map constructed covariantly under rotations. Then, for every rotation R_ϕ about OM ,

$$\mathcal{A}_{\mathcal{C},M}(R_\phi \hat{n}) = \mathcal{A}_{\mathcal{C},M}(\hat{n}). \quad (1)$$

For a scalar map, in spherical harmonics with polar axis along $\hat{e}_r$,

$$a_{\ell m} = 0 \quad (m \neq 0), \quad (2)$$

and every covariantly extracted nonzero dipole vector satisfies

$$\mathbf{d}_A \parallel \pm \hat{e}_r, \quad \mathbf{d}_A \times \mathbf{d}_B = 0. \quad (3)$$

Proof. The subgroup fixing both O and M is the $SO(2)$ of rotations about OM . Each such rotation leaves $\mathcal{C}$, M , and a radial observer velocity invariant. Rotation covariance therefore gives Eq. (1). A scalar function invariant under this $SO(2)$ has only $m = 0$ harmonics in the aligned frame. Any vector extracted covariantly from the same axisymmetric map must itself be invariant under the stabilizer, hence lie on the axis. $\square$

The theorem fixes an *unoriented axis*, not a sign or amplitude. A transverse observer velocity breaks the hypothesis and must be modeled separately. Likewise, anisotropic selection, multiple centers, or non-spherical structure can generate $m \neq 0$ power without contradicting the group-theoretic statement.

II. THREE LOCAL AMPLITUDE LAWS

The axis is fixed by symmetry; amplitudes require a physical channel. Three local expansions are useful because they separate channels without choosing a cosmological field equation.

a. Integrated scalar. In a normal neighborhood of M , let $\gamma_{\hat{n}}(u) = \exp_M(u\hat{n})$ be a unit-speed geodesic and define

$$\begin{aligned} \mathcal{O}_F(s, \hat{n}) &= \int_0^s F(\gamma_{\hat{n}}(u)) du, \\ \mathcal{O}_F^- &= \frac{\mathcal{O}_F(s, \hat{n}) - \mathcal{O}_F(s, -\hat{n})}{2}. \end{aligned} \quad (4)$$

A covariant Taylor expansion gives

$$\mathcal{O}_F^-(s, \hat{n}) = \frac{s^2}{2} \nabla F(M) \cdot \hat{n} + O(s^4). \quad (5)$$

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Thus $\nabla F(M) = 0$ removes the leading local dipole of this channel.

b. Number counts. Let ν be the effective spatial selection density of a catalogue. In geodesic polar coordinates about M , the volume Jacobian is $J = u^2[1 - \frac{1}{6}\text{Ric}_{ij}n^in^ju^2 + O(u^3)]$. The Ricci correction is even under $\hat{n} \rightarrow -\hat{n}$, so

$$\left(\frac{dN}{d\Omega}\right)^- = \frac{s^4}{4} \nabla \nu(M) \cdot \hat{n} + O(s^6), \quad (6)$$

and, for $\nu(M) > 0$ with a monopole-dominated count,

$$\mathcal{D}_N(s) = \frac{3s}{4} |\nabla \ln \nu(M)| + O(s^3). \quad (7)$$

These are local-depth relations; a deep flux-limited catalogue requires its full selection and propagation integrals.

c. Endpoint-potential redshift. Suppose, for the channel under consideration, that the redshift of a specified emitted line is represented by a positive endpoint scalar N ,

$$\mathcal{Z} \equiv \ln(1+z) = \ln \frac{N(M)}{N(x_{\text{em}})}. \quad (8)$$

Sampling equal spatial depths in opposite directions gives

$$\mathcal{Z}^-(s, \hat{n}) = -s \nabla \ln N(M) \cdot \hat{n} + O(s^3). \quad (9)$$

For a radial field, $\nabla \ln N(M) = g_o \hat{e}_r$ with $g_o = N'(r_o)/N(r_o)$.

These laws have different depth dependence. A kinematic Doppler contribution is independent of s ; the endpoint-potential odd part scales as s ; and the integrated-scalar odd part scales as s^2 . For an apparent redshift slope $H_{\text{app}} = c\mathcal{Z}/s$ with isotropic leading term, the corresponding fractional behaviors scale respectively as s^{-1} , s^0 , and s^1 .

III. WHAT THE COMMON AXIS DOES NOT IMPLY

Collinearity does not imply equal signs or nonzero amplitudes. Different observables can be parallel or antiparallel to the same axis, and a zero residual dipole remains compatible with the hypothesis.

There is also an exact counterexample to a frequently tempting inference. Consider a centered static spherical emitting surface of radius R_s , uniform emitted blackbody temperature T_s , and collisionless propagation preserving phase-space occupation. If Eq. (8) governs the stationary redshift, then every ray from that surface gives

$$T_{\text{obs}}(\hat{n}) = T_s \frac{N(R_s)}{N(M)}, \quad (10)$$

which is independent of direction even for an off-center observer in a radial potential. A radial gradient therefore does not, by itself, generate a temperature dipole from a centered isothermal background. The independently measured aberration and modulation of CMB anisotropies provide a separate test of the kinematic interpretation of the observed CMB dipole [7].

IV. A DIRECT FALSIFICATION PROTOCOL

A single-center interpretation should be tested on residuals, in this order:

1. define each observable, radial selection, and evolution model before inspecting the desired axis;
2. subtract independently modeled kinematic, instrumental, mask, and local-structure contributions;
3. estimate each residual dipole vector with its full directional covariance and, where possible, its depth dependence;
4. test whether independently established non-kinematic residual dipoles attributed to the same spherical structure are collinear within uncertainty;
5. in the candidate common-axis frame, test residual $m \neq 0$ power and compare the measured depth scaling with Eqs. (5), (7), and (9).

Significant non-collinearity rejects a common single-center origin. Collinearity is only a necessary compatibility condition; it is not by itself evidence that such a center exists.

Current dipole measurements illustrate why the distinction matters. A large CatWISE2020 count dipole has been reported [4], while subsequent analyses have revised its significance or found results compatible with the CMB-inferred velocity in other samples [5, 6]. In Pantheon+SH0ES, Sorrenti, Durrer and Kunz report a low-redshift distance-redshift dipole whose direction differs from the Planck dipole by about 40° ; the detection weakens as the lower redshift cut is raised and is no longer highly significant for cuts of order $z_{\text{cut}} \simeq 0.05$ and above [8]. These measurements concern different observables, selections, and systematics. The theorem above supplies a model-independent cross-check only after non-kinematic residuals are established separately.

V. CONCLUSION

One spherically symmetric center implies one sky axis for an off-center observer. This is an exact consequence of the rotational stabilizer and requires no cosmological field equation. Local amplitudes add a second discriminator: kinematic, endpoint-potential, integrated-scalar,

and count-selection channels carry different depth scalings. The hypothesis therefore makes a simple falsifiable demand. If two independently established non-kinematic residual dipoles are attributed to the same spherical structure, they must be collinear; if they are not, that common single-center interpretation is excluded.

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DATA AVAILABILITY

No new empirical data were generated or analyzed. All derivations are contained in the manuscript.

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- [1] H. Alnes and M. Amarzguoui, CMB anisotropies seen by an off-center observer in a spherically symmetric inhomogeneous universe, *Phys. Rev. D* **74**, 103520 (2006), doi:10.1103/PhysRevD.74.103520.
 - [2] R. R. Caldwell and A. Stebbins, A Test of the Copernican Principle, *Phys. Rev. Lett.* **100**, 191302 (2008), doi:10.1103/PhysRevLett.100.191302.
 - [3] C. Clarkson, B. Bassett, and T. H.-C. Lu, A General Test of the Copernican Principle, *Phys. Rev. Lett.* **101**, 011301 (2008), doi:10.1103/PhysRevLett.101.011301.
 - [4] N. J. Secrest, S. von Hausegger, M. Rameez, R. Mohayaee, S. Sarkar, and J. Colin, A Test of the Cosmological Principle with Quasars, *Astrophys. J. Lett.* **908**, L51 (2021), doi:10.3847/2041-8213/abdd40.
 - [5] M. Bashir, P. Chingangbam, and S. Appleby, The CatWISE2020 Quasar dipole: A Reassessment of the Cosmic Dipole Anomaly, arXiv:2511.00822 (2025).
 - [6] S. Chen *et al.*, Testing the cosmological principle with quasars, *Astron. Astrophys.* **708**, A307 (2026), doi:10.1051/0004-6361/202556955.
 - [7] Planck Collaboration, N. Aghanim *et al.*, Planck 2013 results. XXVII. Doppler boosting of the CMB: Eppur si muove, *Astron. Astrophys.* **571**, A27 (2014), doi:10.1051/0004-6361/201321556.
 - [8] F. Sorrenti, R. Durrer, and M. Kunz, The dipole of the Pantheon+SH0ES data, *J. Cosmol. Astropart. Phys.* **11** (2023) 054, doi:10.1088/1475-7516/2023/11/054.